FUNCTIONAL ANALYSIS - IMM

LIST 3: SEQUENCE SPACES

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1. Prove that ℓ^∞ is a Banach space.

2. Prove c_0 is a closed subspace of ℓ^∞, but c_{00} and ℓ^1 are not closed in ℓ^∞.

3. Let $E = \ell^1$, $E^* = \ell^\infty$ and $N = c_0$ where $N^* = E$. Determine

$$N^\perp = \{\phi \in E : \hat{\phi}(f) = 0 \ \ \forall\, \phi \in N\}$$

and

$$N^{\perp\perp} = \{f \in E^* : \hat{f}(\phi) = 0 \ \ \forall\, \phi \in N^\perp\}.$$

Check that $N^{\perp\perp} \neq N$. (See page 34, lecture 6 for the definitions of $\hat{\phi}$ and $\hat{f}$.)

4. Let ϕ be a linear functional that is *not* bounded on a Banach space $\mathcal{H}$. Prove that

$$\phi(B_1) = \mathbb{C}$$

where $B_1 = \{x \in \mathcal{H} : ||x|| \leq 1\}$ is the closed unit ball $\mathcal{H}$.

Hint: Use the fact that $||\phi|| = \infty$ *to show that for each* $z \in \mathbb{C}$, *we can find some* $x_0 \in B_1$ *with* $\phi(x_0) = z$.