5. Algebra: Exercise sheet number 5

Submit as written homework the solutions to exercises (1), (5). The other exercises will be discussed in the tutorials (and you should prepare for this discussion).

(1) Show that the units in $\mathbb{Z}[i]$ are $\{1, -1, i, -i\}$.
(2) Show: The odd integers with usual addition and multiplication are not a ring.
(3) An element a in a ring R is called nilpotent if there is an $n \in \mathbb{Z}_{>0}$ with $a^n = 0$. Show: if a is nilpotent, then $1 + a$ is a unit.
(4) Show that a commutative ring R with 1 is an integral domain if and only if for all $a, b, c \in R$ with $a \neq 0$, the relation $ab = ac$ implies $b = c$.
(5) Show a subring S of an integral domain R with $1 \in S$ is an integral domain.