

4. Algebra: Exercise sheet number 4

Submit as written homework the solutions to exercises (1), (2), (6). The other exercises will be discussed in the tutorials (and you should prepare for this discussion).

- (1) Which of the following groups are isomorphic

$$\mathbb{Z}_{24}, \mathbb{Z}_4 \times \mathbb{Z}_6, \mathbb{Z}_3 \times \mathbb{Z}_8, S_4?$$

- (2) If G is a group of order p^2q with $p \neq q$ primes, show that G has a nontrivial normal subgroup.
- (3) How many elements of order 5 are contained in a group of order 20?
- (4) Let G be a nonabelian group of order p, q with $p > q$ prime numbers.
- Show G has a normal subgroup.
 - How many q -Sylow subgroups does G have?
 - What can you say about p and q ?
- (5) Classify the groups of order 33.
- (6) Let G be a group of order $p^m r$ with p a prime number and p not dividing r . Let N be a normal subgroup of G of order $p^m l$ with p not dividing l . Show that N contains all p -Sylow subgroups of G .