3. Algebra: Exercise sheet number 3

Submit as written homework the solutions to exercises (1), (2), (5). The other exercises will be discussed in the tutorials (and you should prepare for this discussion).

(1) Let G be a group with $|G| = 16$ acting on a set with X with 33 elements. Does the action of G have a fix point i.e. an element $x \in X$ with $g \cdot x = x$ for all $g \in G$?

(2) Let G be a finite group. Let $A \subset B \subset G$ be two subgroups. Let $N_G(A) := \left\{ g \in G \mid gAg^{-1} = A \right\}$ be the normaliser of A in G. and $Z_G(A) = \left\{ g \in G \mid gag^{-1} = a \forall_{a \in A} \right\}$ the centralizer of A in G. Which of the following is true? Find proof of counterexample.
   (a) $N_G(A) \subset N_G(B)$
   (b) $N_G(A) \supset N_G(B)$
   (c) $Z_G(A) \subset Z_G(B)$
   (d) $Z_G(A) \supset Z_G(B)$
   (e) $Z(A) \subset Z(B)$
   (f) $Z(A) \supset Z(B)$.

(3) Find a subgroup H of S_3 of order 3, i.e. with $|H| = 3$.

(4) Let
$$\sigma := \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix} \in S_9.$$
   Find the cycle decomposition of σ, find the cycle type of σ, and write an element of S_9 which is conjugated to σ.

(5) Show: An r-cycle in S_n is even if and only if r is odd.