

2. Algebra: Exercise sheet number 2

Submit as written homework the solutions to exercises (4), (5), (6). The other exercises will be discussed in the tutorials (and you should prepare for this discussion).

- (1) Let G be a group, let $\phi : G \rightarrow H$ be a surjective homomorphism, let $Z(G)$ be the center of G .
 - (a) Show $\phi(Z(G)) \subset Z(H)$.
 - (b) Is this still true if ϕ is not surjective?
- (2) Let G be a cyclic group of prime order. Determine $\text{Aut}(G)$.
- (3) Let H, K be normal subgroups of a group G with $H \cap K = \{1\}$. Show that $hk = kh$ for all $h \in H, k \in K$.
- (4) Let G be a group and $Z(G)$ its center. Show: if $G/Z(G)$ is cyclic, then G is abelian.
- (5) Prove that every subgroup of a cyclic group is cyclic.
- (6) A subgroup H of a group G is called characteristic subgroup if $\phi(H) = H$ for all automorphisms ϕ of G . Show that characteristic subgroups are normal subgroups.