

1. Algebra: Exercise sheet number 1

Submit as written homework the solutions to exercises (4), (5), (6). The other exercises will be discussed in the tutorials (and you should prepare for this discussion).

- (1) Let H_1, H_2 be subgroups of a group G . Show $H_1 \cap H_2$ is a subgroup of G .
- (2) Let G be a group and H, K normal subgroups. Then $H \cap K$ is a normal subgroup of G .
- (3) Let A, B be subgroups of a finite group G and assume that $|A|$ and $|B|$ have no common factors. How many elements does $A \cap B$ have? Prove your answer.
- (4) Let G be a group, $H \subset G$ a subgroup of finite index $[G : H]$. A right coset of H in G is a set of the form $Hg = \{hg \mid h \in H\}$. Show that the number of right cosets of H in G is $[G : H]$.
- (5) Let G be a finite group, and H a subgroup.
 - (a) If $[G : H] = 2$, then H is a normal subgroup of G .
 - (b) Give an example of a subgroup of index 3 of a group, which is not normal.
- (6) Let G be a group. The center $Z(G)$ of G is
$$Z(G) := \{g \in G \mid ga = ag, \forall a \in G\},$$
 - (a) Show $Z(G)$ is a normal subgroup of G ,
 - (b) Show $Z(G)$ is commutative.
 - (c) What is the center of S_3 ?
- (7) Let G be a group, let $Z(G)$ be the center of G . Let $N \subset Z(G)$ be a subgroup. Show N is a normal subgroup of G .